[13.39] Let V be an n-dimensional vector space. Let V  be the vector space of -valent tensors on V, where V\* is the dual vector space of V. Show that a linear transformation *T* on V induces a linear transformation  on V where , the transpose of the inverse of *T*.



Solution. We must first show that :

Let  where  and .

Recall that . Therefore



.

Also, . So, for example,  is a sum of vectors in . Therefore

,

and thus

 ✔

To show that T is linear, let *P* and *Q* be -valent tensors, ** a scalar, and *R* = *P* + *Q*. Because the tensor product is multilinear,





and



 ✔